What Roberto taught me

Franco Blanchini [1]

[1]Dipartimento di Scienze Matematiche, Informatiche e Fisiche
Università degli Studi di Udine

SIDRA, September 2017, Milano

Premise of the talk

Premise of the talk

- Roberto's work on parametric uncertainty;

Premise of the talk

- Roberto's work on parametric uncertainty;
- Why I consider him my teacher;

Premise of the talk

- Roberto's work on parametric uncertainty;
- Why I consider him my teacher;
- Some technical results;

Premise of the talk

- Roberto's work on parametric uncertainty;
- Why I consider him my teacher;
- Some technical results;
- Fundamental messages!

Meeting Roberto

Torino 1989: I was giving a talk ....
"I do not think that this work is promising because ...."

Meeting Roberto

Torino 1989: I was giving a talk ....
"I do not think that this work is promising because ...."

Message: Be fair and sincere in expressing your opinion, in a constructive manner.

Meeting Roberto

Torino 1989: I was giving a talk ....
"I do not think that this work is promising because ...."

Message: Be fair and sincere in expressing your opinion, in a constructive manner.

Kappel 1991: "Kris and I had a hard time understanding what you are saying...."

Meeting Roberto

Torino 1989: I was giving a talk ....
"I do not think that this work is promising because ...."

Message: Be fair and sincere in expressing your opinion, in a constructive manner.

Kappel 1991: "Kris and I had a hard time understanding what you are saying...."

Message: Be clear when you are presenting.....

Meeting Roberto

Torino 1989: I was giving a talk ....
"I do not think that this work is promising because ...."

Message: Be fair and sincere in expressing your opinion, in a constructive manner.

Kappel 1991: "Kris and I had a hard time understanding what you are saying...."

Message: Be clear when you are presenting.....

"You might decide to work in the Kharitonov area ...."

Meeting Roberto

Torino 1989: I was giving a talk ....
"I do not think that this work is promising because ...."

Message: Be fair and sincere in expressing your opinion, in a constructive manner.

Kappel 1991: "Kris and I had a hard time understanding what you are saying...."

Message: Be clear when you are presenting.....

"You might decide to work in the Kharitonov area ...."

Message: Always support "young" people in research.

Kharitonov theorem

Kharitonov theorem

The **interval** polynomial

$$p(s,q) = \sum q_k s^k, \qquad q_k^- \leq q_k \leq q_k^+$$

is robustly Hurwitz if and only if

$$\begin{array}{ccccccccc}
q_0^+ & q_1^+ & q_2^- & q_3^- & q_4^+ & q_5^+ & q_6^- & q_7^- & \dots \\
q_0^- & q_1^+ & q_2^+ & q_3^- & q_4^- & q_5^+ & q_6^+ & q_7^- & \dots \\
q_0^- & q_1^- & q_2^+ & q_3^+ & q_4^- & q_5^- & q_6^+ & q_7^+ & \dots \\
q_0^+ & q_1^- & q_2^- & q_3^+ & q_4^+ & q_5^- & q_6^- & q_7^+ & \dots
\end{array}$$

are Hurwitz.

Kharitonov theorem

The **interval** polynomial

$$p(s,q) = \sum q_k s^k, \qquad q_k^- \leq q_k \leq q_k^+$$

is robustly Hurwitz if and only if

$$\begin{array}{cccccccc}
q_0^+ & q_1^+ & q_2^- & q_3^- & q_4^+ & q_5^+ & q_6^- & q_7^- \dots \\
q_0^- & q_1^+ & q_2^+ & q_3^- & q_4^- & q_5^+ & q_6^+ & q_7^- \dots \\
q_0^- & q_1^- & q_2^+ & q_3^+ & q_4^- & q_5^- & q_6^+ & q_7^+ \dots \\
q_0^+ & q_1^- & q_2^- & q_3^+ & q_4^+ & q_5^- & q_6^- & q_7^+ \dots
\end{array}$$

are Hurwitz.

Message: "Results of this type are a progress in science".

Kharitonov theorem

The **interval** polynomial

$$p(s,q) = \sum q_k s^k, \qquad q_k^- \leq q_k \leq q_k^+$$

is robustly Hurwitz if and only if

$$\begin{array}{cccccccc}
q_0^+ & q_1^+ & q_2^- & q_3^- & q_4^+ & q_5^+ & q_6^- & q_7^- \dots \\
q_0^- & q_1^+ & q_2^+ & q_3^- & q_4^- & q_5^+ & q_6^+ & q_7^- \dots \\
q_0^- & q_1^- & q_2^+ & q_3^+ & q_4^- & q_5^- & q_6^+ & q_7^+ \dots \\
q_0^+ & q_1^- & q_2^- & q_3^+ & q_4^+ & q_5^- & q_6^- & q_7^+ \dots
\end{array}$$

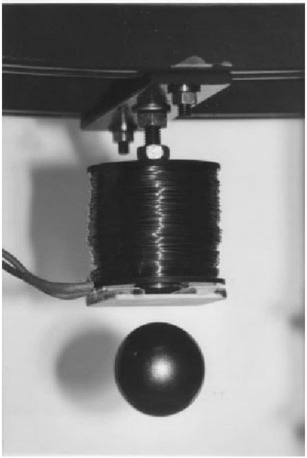
are Hurwitz.

Message: "Results of this type are a progress in science".

"Bob imported from "Soviet Union" this result making it **very clear** that is due to Kharitonov".

Message: Always give credit to other scholars for their work.

Parametric uncertainties



Parametric uncertainties

$$F(s) = \frac{c}{(s^2 - a)(s + b)} \qquad G(s) = \kappa \frac{s + \beta}{s + \alpha}$$

Parametric uncertainties

$$F(s) = \frac{c}{(s^2 - a)(s + b)} \qquad G(s) = \kappa \frac{s + \beta}{s + \alpha}$$

$a = 1400 \pm 400$, $b = 350 \pm 10$ and $c = 1500 \pm 500$
$\kappa = 3000$, $\alpha = 50$, $\beta = 5$.

Parametric uncertainties

$$F(s) = \frac{c}{(s^2 - a)(s+b)} \qquad G(s) = \kappa \frac{s+\beta}{s+\alpha}$$

$a = 1400 \pm 400$, $b = 350 \pm 10$ and $c = 1500 \pm 500$
$\kappa = 3000$, $\alpha = 50$, $\beta = 5$.

To apply Kharitonov theorem write:

$$p(s) = s^4 + \underbrace{[b+\alpha]}_{q_3} s^3 + \underbrace{[b\alpha - a]}_{q_2} s^2 + \underbrace{[\kappa c - a(b+\alpha)]}_{q_1} s + \underbrace{[\kappa c\beta - ab\alpha]}_{q_0}$$

$q_i^- \leq q_i \leq q_i^+$ ... but the uncertain structure is lost

Parametric uncertainties

$$F(s) = \frac{c}{(s^2 - a)(s + b)} \qquad G(s) = \kappa \frac{s + \beta}{s + \alpha}$$

$a = 1400 \pm 400$, $b = 350 \pm 10$ and $c = 1500 \pm 500$
$\kappa = 3000$, $\alpha = 50$, $\beta = 5$.

To apply Kharitonov theorem write:

$$p(s) = s^4 + \underbrace{[b + \alpha]}_{q_3} s^3 + \underbrace{[b\alpha - a]}_{q_2} s^2 + \underbrace{[\kappa c - a(b + \alpha)]}_{q_1} s + \underbrace{[\kappa c \beta - ab\alpha]}_{q_0}$$

$q_i^- \leq q_i \leq q_i^+$ ... but the uncertain structure is lost
Message: "The result is **conservative**".

Value set

Zero exclusion principle.

$$p(s,q) = s^n + q_{n-1}s^{n-1} + \cdots + q_1 s + q_0, \quad q \in \mathscr{Q}$$

$\mathscr{Q}$: generic region in the parameter space.

Value set

Zero exclusion principle.

$$p(s,q) = s^n + q_{n-1}s^{n-1} + \cdots + q_1 s + q_0, \quad q \in \mathcal{Q}$$

$\mathcal{Q}$: generic region in the parameter space.
The value set is

$$V_\omega = \{p(j\omega, q), \;\; q \in \mathcal{Q}\}$$

Value set

Zero exclusion principle.

$$p(s,q) = s^n + q_{n-1}s^{n-1} + \cdots + q_1 s + q_0, \quad q \in \mathcal{Q}$$

$\mathcal{Q}$: generic region in the parameter space.
The value set is

$$V_\omega = \{p(j\omega,q), \;\; q \in \mathcal{Q}\}$$

Theorem

$p(s,q)$ is Hurwitz iff

- *$p(s,q^*)$ is Hurwitz for some $q^* \in \mathcal{Q}$;*
- *$0 \notin V_\omega, \quad$ for all $\;\omega \geq 0$*

Value set

Zero exclusion principle.

$$p(s,q) = s^n + q_{n-1}s^{n-1} + \cdots + q_1 s + q_0, \quad q \in \mathcal{Q}$$

$\mathcal{Q}$: generic region in the parameter space.
The value set is

$$V_\omega = \{p(j\omega, q), \quad q \in \mathcal{Q}\}$$

Theorem

$p(s,q)$ is Hurwitz iff

- *$p(s,q^*)$ is Hurwitz for some $q^* \in \mathcal{Q}$;*
- *$0 \notin V_\omega$, for all $\omega \geq 0$*

Horowitz Qualitative Feedback Theory

Value set

Zero exclusion principle.

$$p(s,q) = s^n + q_{n-1}s^{n-1} + \cdots + q_1 s + q_0, \quad q \in \mathcal{Q}$$

$\mathcal{Q}$: generic region in the parameter space.
The value set is

$$V_\omega = \{p(j\omega, q), \;\; q \in \mathcal{Q}\}$$

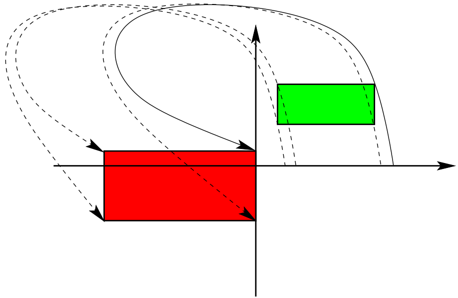
Theorem

$p(s,q)$ is Hurwitz iff

- *$p(s,q^*)$ is Hurwitz for some $q^* \in \mathcal{Q}$;*
- *$0 \notin V_\omega$, for all $\omega \geq 0$*

Horowitz Qualitative Feedback Theory
Message: "Do not forget the past!".

Kharitonov theorem and value set

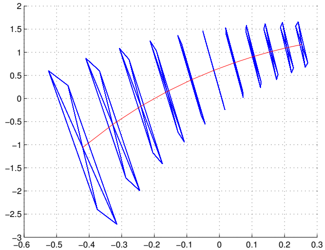


Minnichelli Anagnost Desoer (1989)

Parametric uncertainties: polytopic case

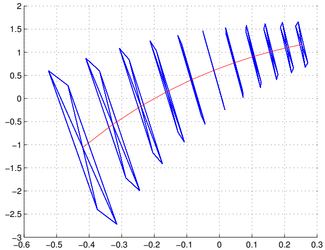
Case 1: a and c uncertain: $a = 1400 \pm 400$, $b = 350$ and $c = 1500 \pm 500$:

$$p(s,a,c) = s^4 + (b+\alpha)s^3 + (\alpha b - a)s^2 + (c\kappa - a(\alpha + b))s + (k\beta - ab\alpha)$$



Case 1: a and c uncertain: $a = 1400 \pm 400$, $b = 350$ and $c = 1500 \pm 500$:

$$p(s,a,c) = s^4 + (b+\alpha)s^3 + (\alpha b - a)s^2 + (c\kappa - a(\alpha + b))s + (k\beta - ab\alpha)$$



Polytope of polynomials: the value set is a polygon.

Parametric uncertainties: non–polytopic

Case 2: a, b and c uncertain: **Multi–affine structure**

$$p(s,a,b,c) = s^4 + (b+\alpha)s^3 + (\alpha b - a)s^2 + (c\kappa - a\alpha - ab)s + (k\beta - ab\alpha)$$

Parametric uncertainties: non–polytopic

Case 2: a, b and c uncertain: **Multi–affine structure**

$$p(s,a,b,c) = s^4 + (b+\alpha)s^3 + (\alpha b - a)s^2 + (c\kappa - a\alpha - ab)s + (k\beta - ab\alpha)$$

Theorem

(Mapping theorem). If the coefficients of $p(s,q)$ are multiaffine functions and

$$\mathcal{Q} = \{Q : q^- \le q \le q^+\}$$

then

$$V_\omega = \{p(j\omega,q), \quad q \in \mathcal{Q}\} \subseteq conv\left\{p(j\omega,q^k), \quad q^k \in vert\mathcal{Q}\right\}$$

Case 2: a, b and c uncertain: **Multi–affine structure**

$$p(s,a,b,c) = s^4 + (b+\alpha)s^3 + (\alpha b - a)s^2 + (c\kappa - a\alpha - ab)s + (k\beta - ab\alpha)$$

Theorem

(Mapping theorem). If the coefficients of $p(s,q)$ are multiaffine functions and

$$\mathcal{Q} = \{Q : q^- \leq q \leq q^+\}$$

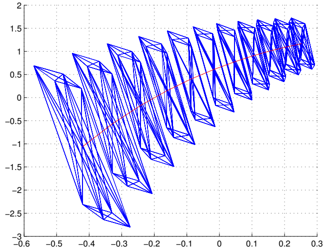
then

$$V_\omega = \{p(j\omega,q), \quad q \in \mathcal{Q}\} \subseteq conv\left\{p(j\omega,q^k), \quad q^k \in vert\mathcal{Q}\right\}$$

Message: "The result concerning polytopes of polynomials are strong, because the assumptions are strong".

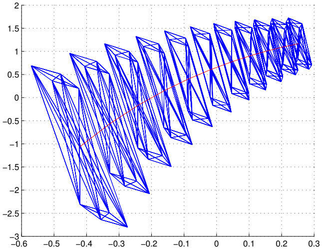
Parametric uncertainties

$a = 1400 \pm 400$, $b = 350 \pm 10$ and $c = 1500 \pm 500$.



Parametric uncertainties

$a = 1400 \pm 400$, $b = 350 \pm 10$ and $c = 1500 \pm 500$.



The “true” value set is inside.

Polytope of polynomials

Polytope of polynomials

- Kharitonov's Theorem is a "vertex result".

Polytope of polynomials

- Kharitonov's Theorem is a "vertex result".
- A polytope of polynomials is robustly stable iff the edges are robustly stable (Bartlet Hollot Lin).

Polytope of polynomials

- Kharitonov's Theorem is a "vertex result".
- A polytope of polynomials is robustly stable iff the edges are robustly stable (Bartlet Hollot Lin).
- An edge of polynomial is not in general robustly stable if the extreme point are stable. Those edges for which this implication holds are the so-called convex directions (Rantzer).

Polytope of polynomials

- Kharitonov's Theorem is a "vertex result".
- A polytope of polynomials is robustly stable iff the edges are robustly stable (Bartlet Hollot Lin).
- An edge of polynomial is not in general robustly stable if the extreme point are stable. Those edges for which this implication holds are the so-called convex directions (Rantzer).
- For special polynomials (e.g. 1-norm ball) you can check only 8 vertices (Barmish, Hollot, Tempo and Kang).

Polytope of polynomials

- Kharitonov's Theorem is a "vertex result".
- A polytope of polynomials is robustly stable iff the edges are robustly stable (Bartlet Hollot Lin).
- An edge of polynomial is not in general robustly stable if the extreme point are stable. Those edges for which this implication holds are the so-called convex directions (Rantzer).
- For special polynomials (e.g. 1-norm ball) you can check only 8 vertices (Barmish, Hollot, Tempo and Kang).
- For other stability regions (unit circle) Karitonov-like conditions do not hold in general. Those regions for which they do are called Kharitonov regions (Rantzer).

Polytope of polynomials

- Kharitonov's Theorem is a "vertex result".
- A polytope of polynomials is robustly stable iff the edges are robustly stable (Bartlet Hollot Lin).
- An edge of polynomial is not in general robustly stable if the extreme point are stable. Those edges for which this implication holds are the so-called convex directions (Rantzer).
- For special polynomials (e.g. 1-norm ball) you can check only 8 vertices (Barmish, Hollot, Tempo and Kang).
- For other stability regions (unit circle) Karitonov-like conditions do not hold in general. Those regions for which they do are called Kharitonov regions (Rantzer).
- Robust root locus (Barmish and Tempo).

Main messages

Main messages

Message: "I like working in the Kharitonov area: there is a lot if competition because there is a lot of interest".

Main messages

Message: “I like working in the Kharitonov area: there is a lot if competition because there is a lot of interest”.

Message: “Be fair in the citations”.

Main messages

Message: "I like working in the Kharitonov area: there is a lot if competition because there is a lot of interest".

Message: "Be fair in the citations".

(Brayton, Tong, Molchanov, Pyatniskii, Larin, Kucera, Sznaier, Propoi, Barabanov, Meilachs ...)

Main messages

Message: "I like working in the Kharitonov area: there is a lot if competition because there is a lot of interest".

Message: "Be fair in the citations".

(Brayton, Tong, Molchanov, Pyatniskii, Larin, Kucera, Sznaier, Propoi, Barabanov, Meilachs . . .)

Message: "Do not waste your time on cheap extensions".

Volumetric margin

In a polytope of polynomial

$$p(s,q) = \sum_{i=1}^{m} q_k \; p_k(s), \quad q_k^- \leq q_k \leq q_k^+$$

The uncertainty interval are typically fixed.

Volumetric margin

In a polytope of polynomial

$$p(s,q) = \sum_{i=1}^{m} q_k \; p_k(s), \quad q_k^- \leq q_k \leq q_k^+$$

The uncertainty interval are typically fixed.
Alternative: Volumetric specification: q_k^+, q_k^- free

$$\mathrm{Vol}[\mathscr{Q}] = \prod_{k=1}^{m} (q_k^+ - q_k^-) \leq V$$

$$q_k^- \leq -\varepsilon + \bar{q}_k < \bar{q}_k + \varepsilon \leq q_k^+$$

Volumetric margin

In a polytope of polynomial

$$p(s,q) = \sum_{i=1}^{m} q_k \; p_k(s), \quad q_k^- \leq q_k \leq q_k^+$$

The uncertainty interval are typically fixed.
Alternative: Volumetric specification: q_k^+, q_k^- free

$$\mathrm{Vol}[\mathcal{Q}] = \prod_{k=1}^{m} (q_k^+ - q_k^-) \leq V$$

$$q_k^- \leq -\varepsilon + \bar{q}_k < \bar{q}_k + \varepsilon \leq q_k^+$$

Minimum destabilizing volume (exact computation): Blanchini, Dabbene, Tempo (1998). The idea was very simple but new ...

Parametric area: only polynomials?

- Interval plants

$$\frac{n_0(s) + \sum_k q_k n_k(s)}{d_0(s) + \sum_k q_k d_k(s)}$$

Parametric area: only polynomials?

- Interval plants

$$\frac{n_0(s) + \sum_k q_k n_k(s)}{d_0(s) + \sum_k q_k d_k(s)}$$

- The value set is delimited by circle arcs (Fu).

Parametric area: only polynomials?

- Interval plants

$$\frac{n_0(s)+\sum_k q_k n_k(s)}{d_0(s)+\sum_k q_k d_k(s)}$$

- The value set is delimited by circle arcs (Fu).
- Nyquist envelope for transfer functions (Hollot and Tempo).

Parametric area: only polynomials?

- Interval plants
$$\frac{n_0(s) + \sum_k q_k n_k(s)}{d_0(s) + \sum_k q_k d_k(s)}$$
- The value set is delimited by circle arcs (Fu).
- Nyquist envelope for transfer functions (Hollot and Tempo).
- For Lead-Lad compensators and interval plants robust stability is ensured if 16 plants are stabilized (Barmish, Hollot, Kraus and Tempo).

Parametric area: only polynomials?

- Interval plants

$$\frac{n_0(s) + \sum_k q_k n_k(s)}{d_0(s) + \sum_k q_k d_k(s)}$$

- The value set is delimited by circle arcs (Fu).
- Nyquist envelope for transfer functions (Hollot and Tempo).
- For Lead-Lad compensators and interval plants robust stability is ensured if 16 plants are stabilized (Barmish, Hollot, Kraus and Tempo).
- Robustness margin: can be a discontinuous functions of the problem data (Barmish, Khargonekar, Shi and Tempo) (also, Pritchard and Hinrichsen).

Parametric area: only polynomials?

- Interval plants

$$\frac{n_0(s) + \sum_k q_k n_k(s)}{d_0(s) + \sum_k q_k d_k(s)}$$

- The value set is delimited by circle arcs (Fu).
- Nyquist envelope for transfer functions (Hollot and Tempo).
- For Lead-Lad compensators and interval plants robust stability is ensured if 16 plants are stabilized (Barmish, Hollot, Kraus and Tempo).
- Robustness margin: can be a discontinuous functions of the problem data (Barmish, Khargonekar, Shi and Tempo) (also, Pritchard and Hinrichsen).
- ...I became aware of these results thanks to Roberto!

Parametric area: only polynomials?

- Interval plants
$$\frac{n_0(s) + \sum_k q_k n_k(s)}{d_0(s) + \sum_k q_k d_k(s)}$$
- The value set is delimited by circle arcs (Fu).
- Nyquist envelope for transfer functions (Hollot and Tempo).
- For Lead-Lad compensators and interval plants robust stability is ensured if 16 plants are stabilized (Barmish, Hollot, Kraus and Tempo).
- Robustness margin: can be a discontinuous functions of the problem data (Barmish, Khargonekar, Shi and Tempo) (also, Pritchard and Hinrichsen).
- . . . I became aware of these results thanks to Roberto!

Message: Be aware of the literature and share your knowledge.

Parametric area: future?

Parametric area: future?

Roberto decided to move to the probabilistic approach in 1996.

Parametric area: future?

Roberto decided to move to the probabilistic approach in 1996.

Parametric area: future?

Roberto decided to move to the probabilistic approach in 1996.

Probabilistic bolt: I will protect your fall with probability

$$Pr\{\textbf{Fall is safe}\} = 0.999762$$

Parametric area: future?

Parametric area: future?

Message: I prefer not to stay stubbornly attached to problems that have not been solved after so may years ....

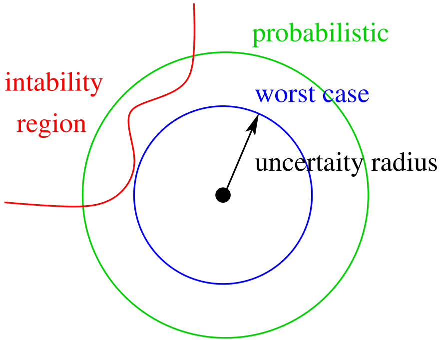
Parametric area: future?

Message: I prefer not to stay stubbornly attached to problems that have not been solved after so may years ....

Message: Probabilistic and worst case analysis are not mutually exclusive. Once you have established a safe guaranteed margin than it is legitimate to ask which is the probability of instability etc. if you violate the bound. (Good point).

Parametric area: future?

Parametric area: future?



Parametric area: future?

Parametric area: future?

Back in 1989, Roberto had predicted that in a few years the research activity on "parametric uncertainties" would have been shading. Basically this is what happened around 2000.

Parametric area: future?

Back in 1989, Roberto had predicted that in a few years the research activity on "parametric uncertainties" would have been shading. Basically this is what happened around 2000.

The spirit of those results is still important. I find Barmish's book a source of inspiration and consult it frequently.

Parametric area: future?

Back in 1989, Roberto had predicted that in a few years the research activity on “parametric uncertainties” would have been shading. Basically this is what happened around 2000.

The spirit of those results is still important. I find Barmish’s book a source of inspiration and consult it frequently.

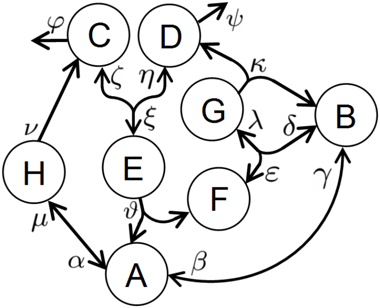
Message: Look around, there are other fields ....

Parametric area: future?

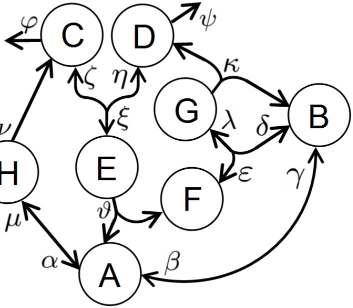
Example: systems biology, the influence matrix

Parametric area: future?

Example: systems biology, the influence matrix



Example: systems biology, the influence matrix



Structural steady-state influence of j on $i \in \{+, -, 0, ?\}$

Parametric area: future?

$$\begin{bmatrix} + & + & + & + & + & + & + & + \\ + & + & + & + & + & + & + & + \\ + & + & + & 0 & + & 0 & + & + \\ 0 & 0 & 0 & + & + & + & + & 0 \\ + & + & + & + & + & + & + & + \\ ? & ? & ? & ? & ? & ? & ? & ? \\ + & + & + & + & + & + & + & + \\ + & + & + & + & + & + & + & + \end{bmatrix}.$$

Parametric area: future?

$$\begin{bmatrix} + & + & + & + & + & + & + & + \\ + & + & + & + & + & + & + & + \\ + & + & + & 0 & + & 0 & + & + \\ 0 & 0 & 0 & + & + & + & + & 0 \\ + & + & + & + & + & + & + & + \\ ? & ? & ? & ? & ? & ? & ? & ? \\ + & + & + & + & + & + & + & + \\ + & + & + & + & + & + & + & + \end{bmatrix}.$$

A “vertex” result has been found
(Giordano, Cuba Samaniego, Franco and Blanchini, Journal of Mathematical Biology, 2016)

The best message

The best message

Question: "Which is the result?"

The best message

Question: "Which is the result?"

My answer: a **theoretical** result is significant if it takes two minutes to explain to a colleague which is the problem and why it is important, and it takes one minute to explain the essential of the result and its novelty.

Conclusions

Conclusions

- This presentation has been limited to what Roberto taught me in Research.

Conclusions

- This presentation has been limited to what Roberto taught me in Research.
- His teaching in life, climbing and editorial activity would require too much time.

Conclusions

- This presentation has been limited to what Roberto taught me in Research.
- His teaching in life, climbing and editorial activity would require too much time.
- I deliberately kept the technical part limited in order to emphasize the essential messages. Details are in

Conclusions

- This presentation has been limited to what Roberto taught me in Research.
- His teaching in life, climbing and editorial activity would require too much time.
- I deliberately kept the technical part limited in order to emphasize the essential messages. Details are in
- R. Tempo and F. Blanchini, "Robustness Analysis with Real Parametric Uncertainty," The Control Handbook, Second Edition (W. S. Levine, Editor), Taylor and Francis 2010.

Conclusions

- This presentation has been limited to what Roberto taught me in Research.
- His teaching in life, climbing and editorial activity would require too much time.
- I deliberately kept the technical part limited in order to emphasize the essential messages. Details are in
- R. Tempo and F. Blanchini, "Robustness Analysis with Real Parametric Uncertainty," The Control Handbook, Second Edition (W. S. Levine, Editor), Taylor and Francis 2010.
- Roberto's activity has always been consistent with all these principles: fairness, originality, significance . . .

Conclusions

Let us follow his route ....