Prospettive e nuove sfide tecnologiche e scientifiche nel settore dell'Automatica

Compressed sensing in multi-agent systems

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Why focus on CS?

- Technique for non-adaptive acquisition, representation, reconstruction of large sparse/compressible data using few measurements [Candes, Romberg & Tao, 2004]

"... the paradigm-busting field in mathematics that's reshaping the way people work with large data sets", 2010

"hottest topic in applied math today", [Candes rewarded with Waterman Prize 2006]

- millions of dollars/euros in grants (CRISP, Reco Samp, SPARCS, PLEASE, C-SENSE, SpaRTaN, ...)
- CS fever (thousands papers) in pure and applied mathematics, information theory, signal processing, circuit design, optical engineering, biomedical imaging, ...)

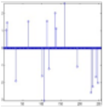
CS in control community

- Fewer papers so far (system identification, optimal control, ...)
- Growing interest from 2010
- See *"A Tutorial on Compressed Sensing and Control Theory: Some Answers and Some Questions"* [Vidyasagar, CDC'2016]

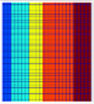
Purpose of these slides

- Review of basic principles of CS (new tool)
- Framework with applications in multi-agent systems
    1. Distributed Compressed Sensing in intelligent networked systems
    2. Identification and inference in social systems (new application)
    3. Sparse optimal control (new field)
- Discuss open questions
- Opportunities for inter-disciplinary work

Sparsity is ubiquitous...



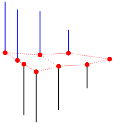
sparsity on elements

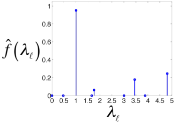


sparsity on singular values

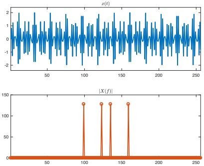
sparsity on eigenvalues

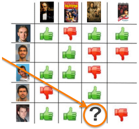
sparsity on tensor singular values

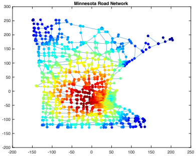


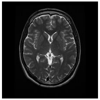


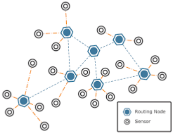
GFT-sparsity











Compressed sensing in a nutshell

Foundational result: k-sparse signal $x \in \mathbb{R}^n$ with $k \ll n$

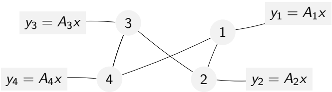
1. Take $m \ll n$ measurements
$$y = Ax$$
   - fat random matrix $A \in \mathbb{R}^{m \times n}$
   - measurements $y \in \mathbb{R}^m$
2. Reconstruction possible w.h.p. using $m = O(k \log(n/k))$
$$\widehat{x}_{BP} = \text{argmin} \|x\|_1 \text{ s.t. } y = Ax$$
3. Many low-complexity iterative algorithms (hundreds of papers)

Nonadaptive compression useful when

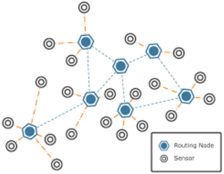
- measurements are expensive, but some computational capabilities available
- data must be processed and communicated efficiently (dimensionality reduction)

Apply CS in multi-agent systems where each individual can

- sense its immediate environment
- store limited amount of information
- perform a low number of operations
- communicate under some constraints



Applications: robotic systems, sensor networks for surveillance, indoor localization and monitoring



Classical framework: distributed compressed sensing

- distributed compressed acquisition in a sensor network;
- centralized reconstruction from few linear measurements
- drawbacks: energy utilization, delays, robustness, privacy

New trend: distributed reconstruction (no fusion center) [Mota&al. 2012, Patterson&al. 2014, Ravazzi&al. 2015, Ravazzi&al. 2016]

(?) distribute the reconstruction task over the network

(?) design distributed, low-complex, low-memory and randomized algorithms with theoretical guarantees

(?) cope with agents' limited computational power and memory

(?) minimize the total number of transmissions

(?) deal with nonlinear measurements: quantization, noise, ...

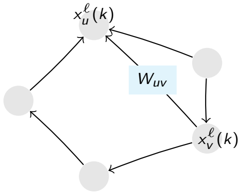
Assumptions:

- Population of individuals
- Discussion on several topics
- Individuals have opinions
- People interact
- Opinions evolve

Mathematical model:

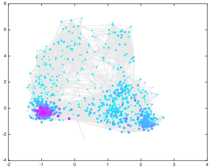
$\mathcal{G} = (\mathcal{V}, \mathcal{E}, W) \leftrightarrow$ Social network

- $v \in \mathcal{V} \leftrightarrow$ agents
- $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V} \leftrightarrow$ interactions
- $W \in \mathbb{R}^{\mathcal{V} \times \mathcal{V}} \leftrightarrow$ influences
- $W_{uv} = 0$ if $(u, v) \notin \mathcal{E}$
- $x_v^\ell(k) \in \mathbb{R} \leftrightarrow$ opinions on issue ℓ
- $x^\ell(k+1) = f_W(x^\ell(k))$



Assumptions:

- W sparse $\leftrightarrow$ people influenced by few friends
- W low rank $\leftrightarrow$ few communities



Facebook network

Main challenges:

(?) Estimate W from partial observations of opinions
[A. Scaglione & al. 2016], [C. Ravazzi, R. Tempo, F. Dabbene, 2016]

(?) Detect communities from compressed/sparse measurements
[C. Ravazzi & al. 2015]

(?) Detect stubborn, influential agents

(?) Estimate spectral properties from compressed/sparse measurements
[J. Hendrickx & al. 2017]

Applications: Trust-based recommending systems (Amazon, Apple, Booking.com, ...)

Assumptions:

- Population of individuals
- Individuals have opinions
- People interact and opinions evolve

Main challenges: sparse optimal controls (acting on few nodes/edges only) to lead

(?) the dynamics towards a desired pattern

[M. Bongini, M. Caponigro, M. Fornasier, B. Piccoli, and E. Trelat, 2012-2017]

(?) qualitative changes to the limit profile (e.g., merge clusters together)

(?) quantitative changes to some observable (e.g., average opinion, target nodes) [Yildiz & al. 2013]

Take-home messages:

- Sparse models useful for estimation/inference/learning problems in multi-agent systems
- New notion of sparsity for data living on irregular structures using graph based representation
- Framework with numerous applications in multi-agents systems
- Still many open problems
- Opportunities for inter-disciplinary work (graph theory, linear algebra, probability, signal processing, control theory, detection and estimation theory, optimization, machine learning, ...)

Sparsity

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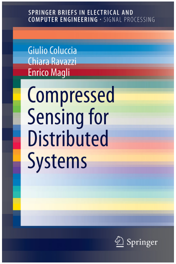

- Introduction to Distributed Compressed Sensing
- Centralized/distributed reconstruction algorithms
- Practical implementations, comparative analysis
- Open theoretical issues

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