National Research Council of Italy

Institute of Electronics Computer and Telecommunication Engineering

SIDRA 2017
Sessione speciale in memoria di Roberto Tempo

Robustezza probabilistica

ovvero come affrontare le incertezze con ottimismo

Fabrizio Dabbene

2007 IFAC Fellow

for his contributions to the analysis and control of uncertain systems, and for pioneering the probabilistic approach to robustness

Outline – Roberto's legacy

Probabilistic robustness:
i.e approaching uncertainty from the optimistic side

1. Motivations: limits of deterministic approach
2. Randomized algorithms for probabilistic robustness analysis
3. Iterative algorithms for probabilistic design
4. New directions

Outline – Roberto’s legacy

1.

Motivations: limits of deterministic approach

The late 90's: Limits of robust paradigm

- In the late 90's research on robust control had reached a good level of maturity
- At the same time, the intrinsic limits of this paradigm became clear
- Conservatism: to guarantee performance for all uncertainties, need to account for uncertainty instances which will never occur
- Computational complexity: scale badly with respect to the number of uncertain parameters

Deterministic Uncertainty

- Deterministic uncertainty: pessimistic viewpoint

"If there is a fifty-fifty chance that something can go wrong, 9 out 10 times it will"
Paul Harvey

Probabilistic Uncertainty

- Probabilistic uncertainty: optimistic viewpoint

"Don't assume the worst-case scenario. It's emotionally draining and probably won't happen anyway"
Anonymous

Outline – Roberto’s legacy

2.

Randomized algorithms for probabilistic robustness analysis

The spark – Probabilistic Robustness

Proceedings of the 35th
Conference on Decision and Control
Kobe, Japan • December 1996

FA09 10:10

Probabilistic Robustness Analysis: Explicit Bounds for the Minimum Number of Samples†

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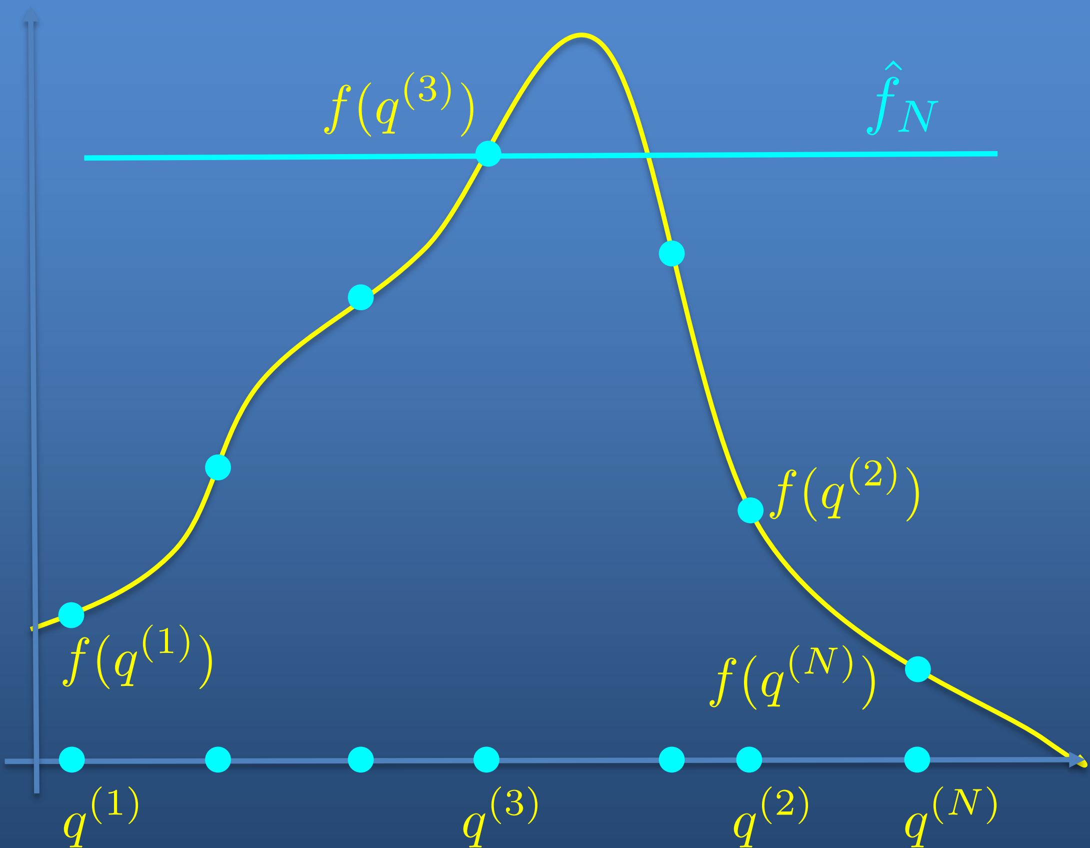
Abstract

In this paper, we study robustness analysis of control systems affected by bounded uncertainty. Motivated by the difficulty to perform this analysis when the uncertainty enters into the plant coefficients in a nonlinear fashion, we study a probabilistic approach. In this setting, the uncertain parameters q are random variables bounded in a set Q and described by a multivariate density function $f(q)$. We then ask the following question: Given a performance level, what is the probability that this level is attained? The main content of this paper is to derive *explicit bounds* for the number of samples required to estimate this probability with a certain accuracy and confidence apriori specified. It is shown that the number obtained is inversely proportional to these thresholds and it is much smaller than that of classical results. Finally, we remark that the same approach can be used to study several problems in a control system context. For example, we can evaluate the worst-case

are concerned, only sufficient conditions are available. If robustness is formulated in state space, stability considerations require to analyze the characteristic polynomial whose coefficients are multiaffine functions of the entries of the state space matrix. Even for this special nonlinearity, no easily computable necessary and sufficient condition is available and one can construct examples for which the value set approximation provided by the so-called Mapping Theorem [4] is highly conservative. When stability of a polytope of matrices is under attention, the iterative linear programming approach developed in [5] leads to necessary and sufficient conditions but the computational complexity of this algorithm may be high for the number of "branching operations" required. To make things even worse from the computational point of view, a number of negative results in terms of NP-hardness have been recently proved; e.g., see [6] and [7].

Motivated by this gloomy picture, in this paper we take

Worst-case performance



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Motivated by this gloomy picture, in this paper we take

- generate samples

$$q^{(1)}, \ldots, q^{(N)}$$

- evaluate

$$f(q^{(1)}), \ldots, f(q^{(N)})$$

- empirical maximum $\hat{f}_N$

$$\mathbb{P}_q\left\{f(q) > \hat{f}_N\right\} \leq \varepsilon$$

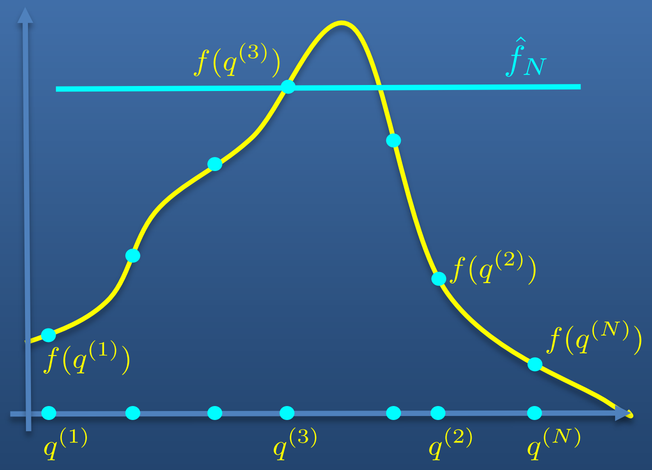
Worst-case performance

- If we choose a number of samples (sample complexity)

$$N \geq \frac{1}{\varepsilon} \ln \frac{1}{\delta}$$

- Then $\mathbb{P}_q \left\{ f(q) > \hat{f}_N \right\} \leq \varepsilon$ holds with probability $\geq \delta$

- Randomized algorithm (RA)
- Explicit bounds on sample complexity
- Possibility of RA failure

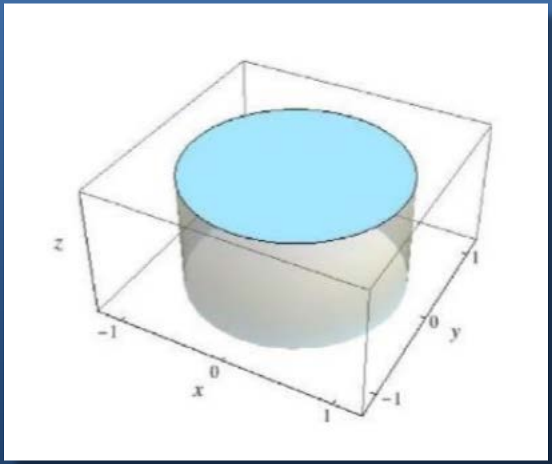


Random Uncertainty

- Random vector (matrix) q
- q bounded in support set $\mathcal{Q} \subset \mathbb{R}^m$
- Multivariate (uniform) pdf associated to $q \in \mathcal{Q}$

multivariate uniform pdf

$$\mathcal{U}[\mathcal{Q}] \doteq \begin{cases} \frac{1}{\mathrm{vol}(\mathcal{Q})} & \text{if}\, q \in \mathcal{Q} \\ 0 & \text{otherwise} \end{cases}$$



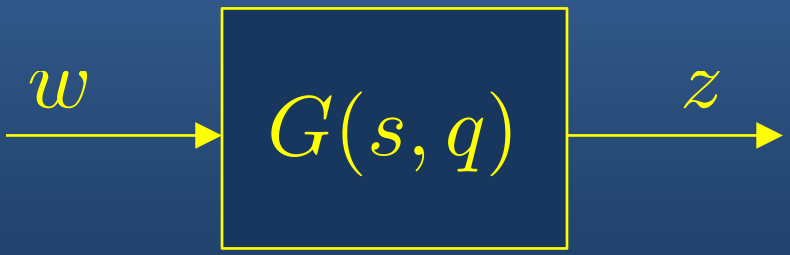
System Constraints

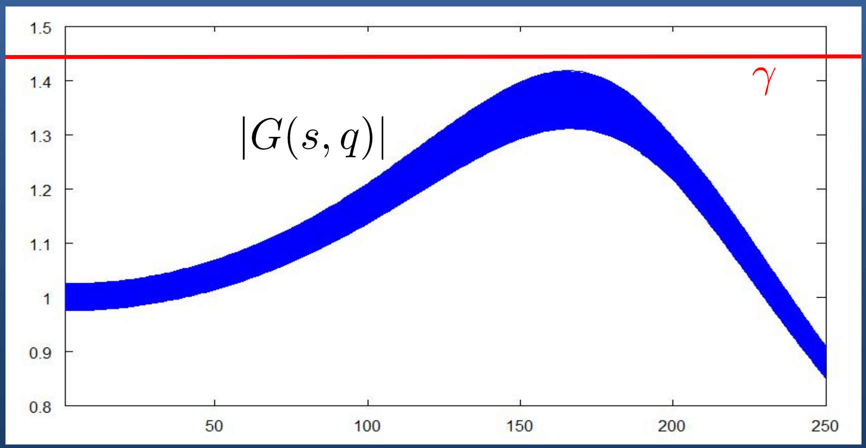
- Define system constraints

$$f(q) \leq 0$$

$f(q) : \mathcal{Q} \to \mathbb{R}$ is a measurable function

- Example: $\mathcal{H}_\infty$ performance
- $G(s,q)$ is stable and $f(q) = \|G(s,q)\|_\infty - \gamma$



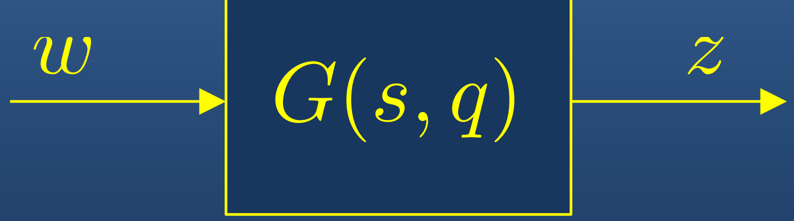


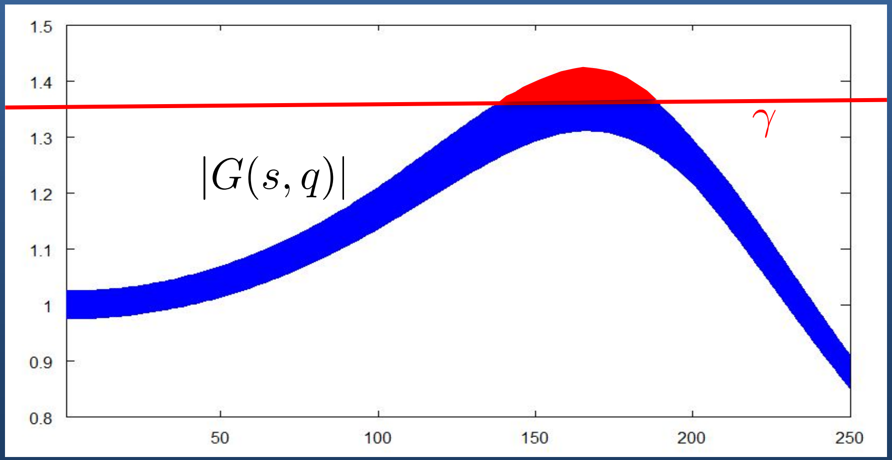
Small Violation and Reliability

- Sufficiently small violation (within ϵ) may be acceptable

$$\mathrm{V}_f = \mathrm{Prob}\{q \in \mathcal{Q} : \; f(q) > 0\} \leq \epsilon$$

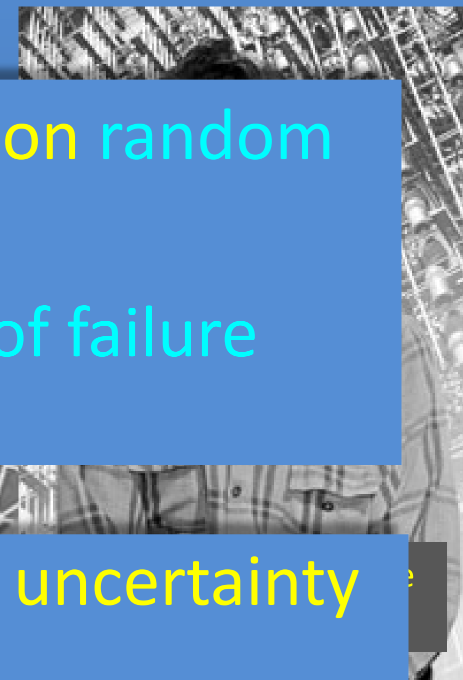
where $\epsilon \in (0,1)$ is a probabilistic parameter





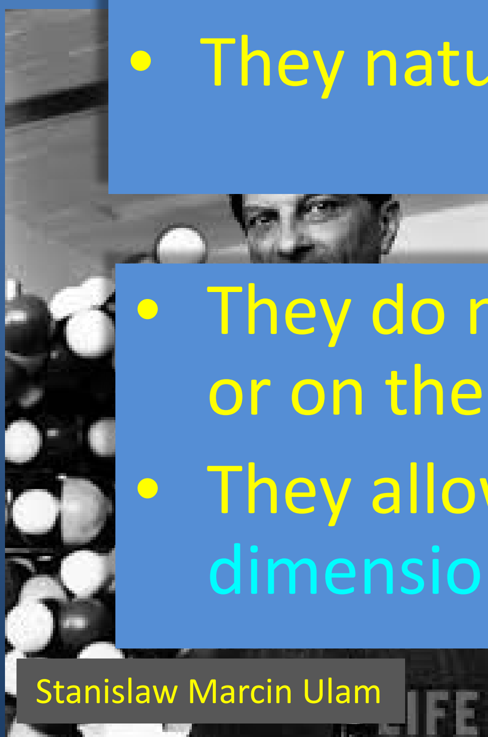
Randomized algorithms

Monte Carlo method invented in the '40s during Manhattan project



- Randomized algorithms are based on random sample extractions
- They naturally entail a probability of failure

- They do not depend on size of the uncertainty or on the way it enters the system
- They allow to break the curse of dimensionality



Probabilistic Robustness Analysis

2218 IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 45, NO. 12, DECEMBER 2000

Randomized Algorithms for Probabilistic Robustness with Real and Complex Structured Uncertainty

Giuseppe C. Calafiore, Fabrizio Dabbene, and Roberto Tempo, *Fellow, IEEE*

***Abstract*—In recent years, there has been a growing interest in developing randomized algorithms for probabilistic robustness of uncertain control systems. Unlike classical worst case methods, these algorithms provide probabilistic estimates assessing, for instance, if a certain design specification is met with a given probability. One of the advantages of this approach is that the robustness margins can be often increased by a considerable amount, at the expense of a small risk. In this sense, randomized algorithms may be used by the control engineer together with standard worst case methods to obtain additional useful information.**

The applicability of these probabilistic methods to robust control is presently limited by the fact that the sample generation is feasible only in very special cases which include systems affected by real parametric uncertainty bounded in rectangles or spheres. Sampling in more general uncertainty sets is generally performed through overbounding, at the expense of an exponential rejection rate.

In this paper, randomized algorithms for stability and performance of linear time invariant uncertain systems described by a general M-Δ configuration are studied. In particular, efficient polynomial-time algorithms for uncertainty structures Δ consisting of an arbitrary number of full complex blocks and uncertain parameters are developed.

***Index Terms*—Random matrices, randomized algorithms, robust control, uncertainty.**

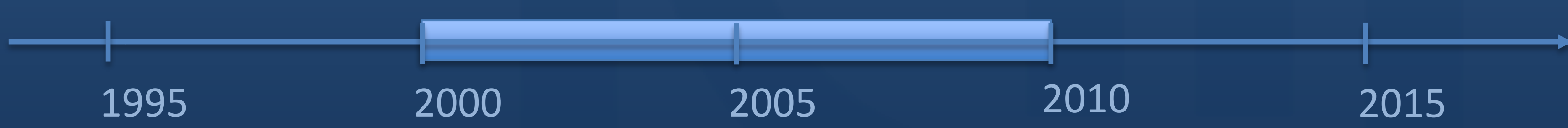
complexity issues of feedback system; see [5], [7], [15], [28], and [30]. The contribution of these papers is to demonstrate that several problems in linear robust control are NP-hard, which in turn implies that they are not practically tractable, unless the number of uncertainties entering into the feedback system is very limited. To avoid this drawback, many other contributions attacked the same problem following a parallel line of research, with the goal of computing upper and lower bounds (instead of the "true" value) of the robustness margin for very general feedback configurations. In other words, the focal point of these papers is to develop either necessary or sufficient conditions for robust stability and performance. The nice feature of these bounds is that their evaluation generally requires the solution of convex programs which can be easily performed, for example, by means of interior point methods [6]. However, the issue of conservatism is still present.

In order to overcome these difficulties, a different paradigm has recently emerged. This new paradigm studies uncertain feedback systems from a probabilistic point of view; see, e.g., [3], [13], [25], [31], [41], and [43]; additional references can be found in the survey papers [38] and [39]. This framework is not alternative to worst case robust control, but it provides useful and complementary information to the control engineer.

Outline – Roberto’s legacy

3.

Sequential algorithms for probabilistic design



Probabilistic design

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Systems & Control Letters 43 (2001) 343–353

SYSTEMS & CONTROL LETTERS

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Probabilistic robust design with linear quadratic regulators

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Received 20 June 2000; received in revised form 1 March 2001

Abstract

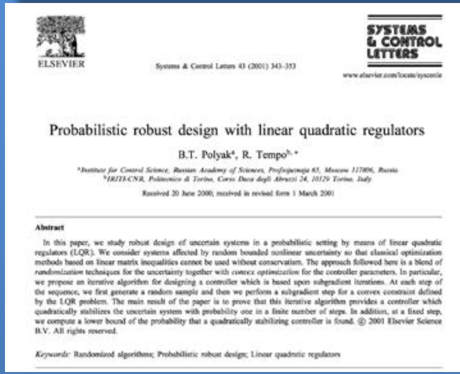
In this paper, we study robust design of uncertain systems in a probabilistic setting by means of linear quadratic regulators (LQR). We consider systems affected by random bounded nonlinear uncertainty so that classical optimization methods based on linear matrix inequalities cannot be used without conservatism. The approach followed here is a blend of *randomization* techniques for the uncertainty together with *convex optimization* for the controller parameters. In particular, we propose an iterative algorithm for designing a controller which is based upon subgradient iterations. At each step of the sequence, we first generate a random sample and then we perform a subgradient step for a convex constraint defined by the LQR problem. The main result of the paper is to prove that this iterative algorithm provides a controller which quadratically stabilizes the uncertain system with probability one in a finite number of steps. In addition, at a fixed step, we compute a lower bound of the probability that a quadratically stabilizing controller is found.

Keywords: Randomized algorithms; Probabilistic robust design; Linear quadratic regulators

Probabilistic design

B.T. Polyak, R. Tempo SCL 2001

Probabilistic design



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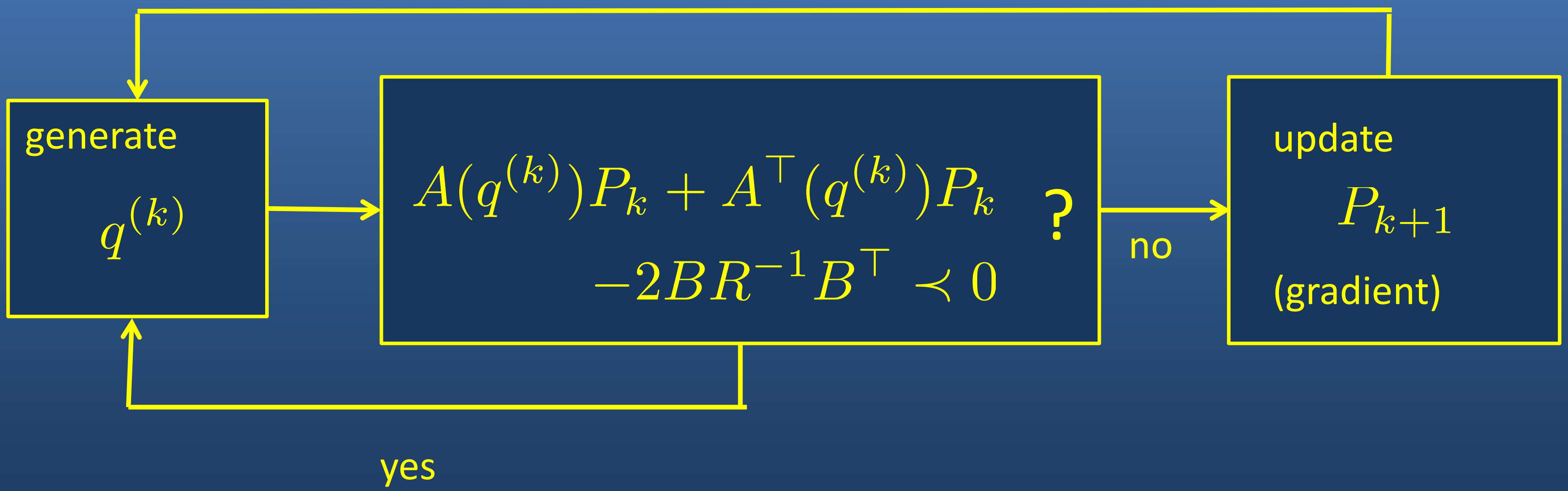
Keywords: Randomized algorithms; Probabilistic robust design; Linear quadratic regulators

$$\dot{x} = A(q)x + Bu, \quad x(0) = x_0$$

$$P \succ 0 \quad :$$

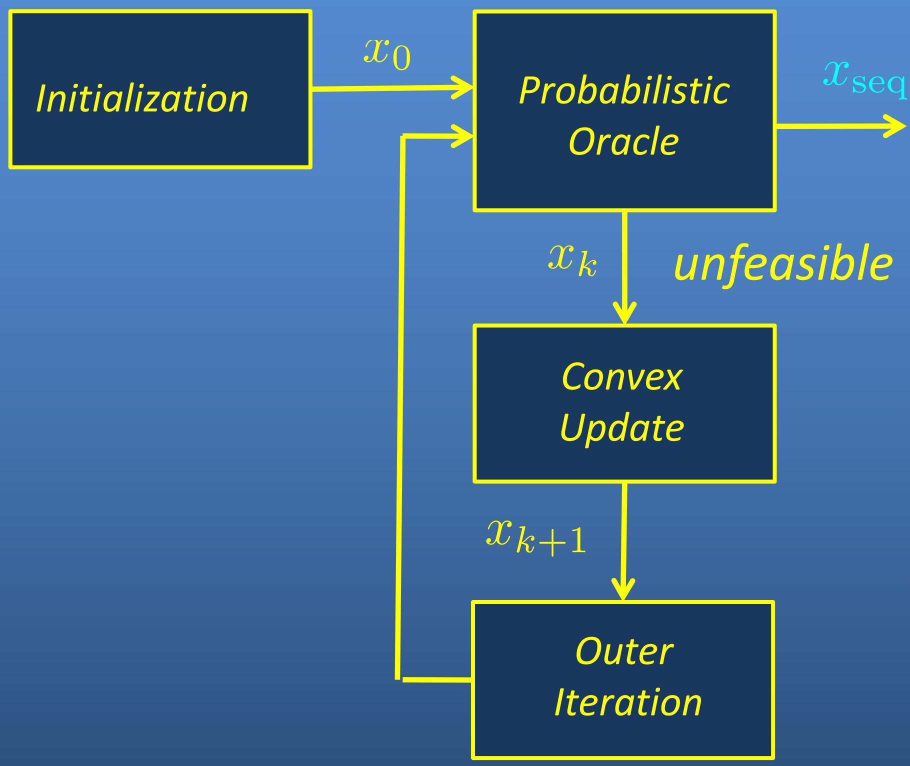
$$A(q)P + A^\top(q)P - 2BR^{-1}B^\top \prec 0 \quad \forall q \in \mathbb{Q}$$



Sequential Algorithms

- Based on a combination of randomization and convex programming

- At each step k, a candidate solution x_k is
  1. checked via sampling
  2. updated exploiting convexity



The years 2000 – Roberto's work

- Switching systems
- Networks
- Circuit analysis
- Fixed-order control design
- UAV control

IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 49, NO. 6, JUNE 2004

754 IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 50, NO. 6, JUNE 2005

IEEE TRANSACTIONS ON NEURAL NETWORKS, VOL. 16, NO. 5, SEPTEMBER 2005 1229

Control Engineering Practice 17 (2009) 974–983

Contents lists available at ScienceDirect

Control Engineering Practice

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Randomization-based control design for Mini-UAVs

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ABSTRACT

The area of research on probabilistic and randomized methods for analysis and design of uncertain systems is fairly recent and is focused both on algorithmic as well as theoretical developments. In this paper a framework for randomization-based control design is presented and applied to a Mini-UAV platform. The proposed approach makes use of random search and uncertainty randomization for controller synthesis and probabilistic robustness analysis. Several structured uncertain parameters, related to the plant and to the operating conditions, are taken into account to design a robust flight control system. A selection criterion, based on estimated probability and its degradation function, is proposed in order to match stability and performance metrics fulfillment. Computational issues associated to the specific application, integration of a priori domain knowledge and human designer interaction with automated design are also addressed.

1. Introduction

In the last decade, the acronym MAVs (micro-aerial vehicles) has been used to define flying objects characterized by physical size approximately smaller than 6 in, in length, width or height. Recalling a classical definition, see, e.g. Davis (1996), these flying objects can be considered as "aerial robots, as six degrees-of-freedom machines whose mobility can deploy a useful micropay-load to a remote, or otherwise hazardous location, where it may perform any of a variety of missions." The concept of unmanned

and inertial data represent a possible set of uncertainties in plant and environment modeling. Therefore, the design of a robust flight control system which guarantees a suitable level of tolerance to environmental changes and platform manufacturing/modeling inaccuracies plays a key role whenever stability and performance requirements have to be fulfilled (see Lombaerts, Mulder, Voorsluijs, & Decupyere, 2005).

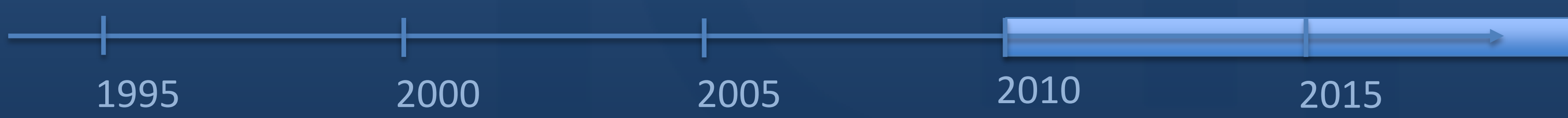
Classical techniques such as, for example, μ-theory, see Zhou, Doyle, and Glover (1996), and linear-parameter varying (LVP), see Lu and Wu (2006) and Fujisaki, Dabbene, and Tempo (2003), have

- In the last decade, the area of randomized algorithms for probabilistic robustness has become a mature field of research
- In the years, many valuable researchers have been attracted to the field
- Probabilistic methods are ready for applications and cross-fertilization with other areas

Outline – Roberto's legacy

4.

New directions



New directions in probabilistic robustness

- K You, R Tempo, L Qiu, “Distributed algorithms for computation of centrality measures in complex networks, IEEE TAC 2017
- E Capello, ,E Punta, F Dabbene, G Guglieri, R Tempo, “Sliding-Mode Control Strategies for Rendezvous and Docking Maneuvers”, JGCD 2017
- M. Lorenzen, FD, R. Tempo and F. Allgower, "Constraint Tightening and Stability in Stochastic Model Predictive Control," IEEE TAC 2017
- S. Formentin, FD, R. Tempo, L. Zaccarian and S. M. Savaresi, "Robust Linear Static Anti-Windup with Probabilistic Certificates," IEEE TAC 2017
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New directions in probabilistic robustness

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IEEE TAC 2017

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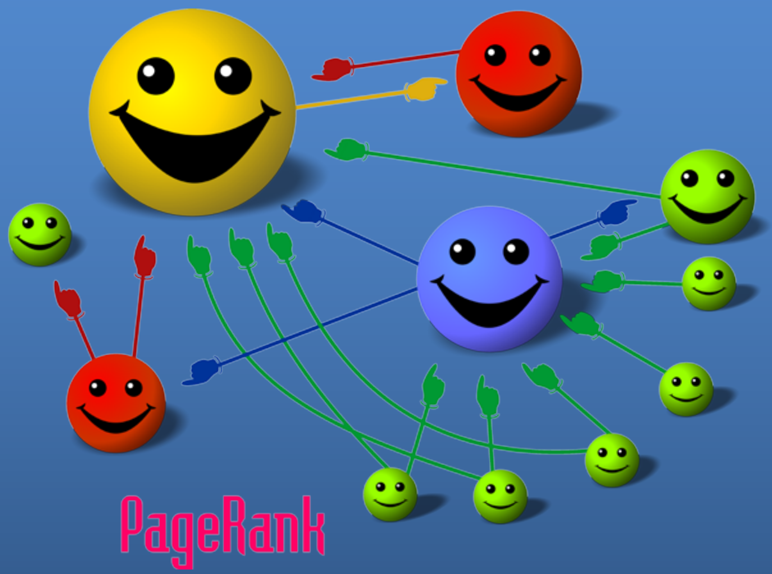
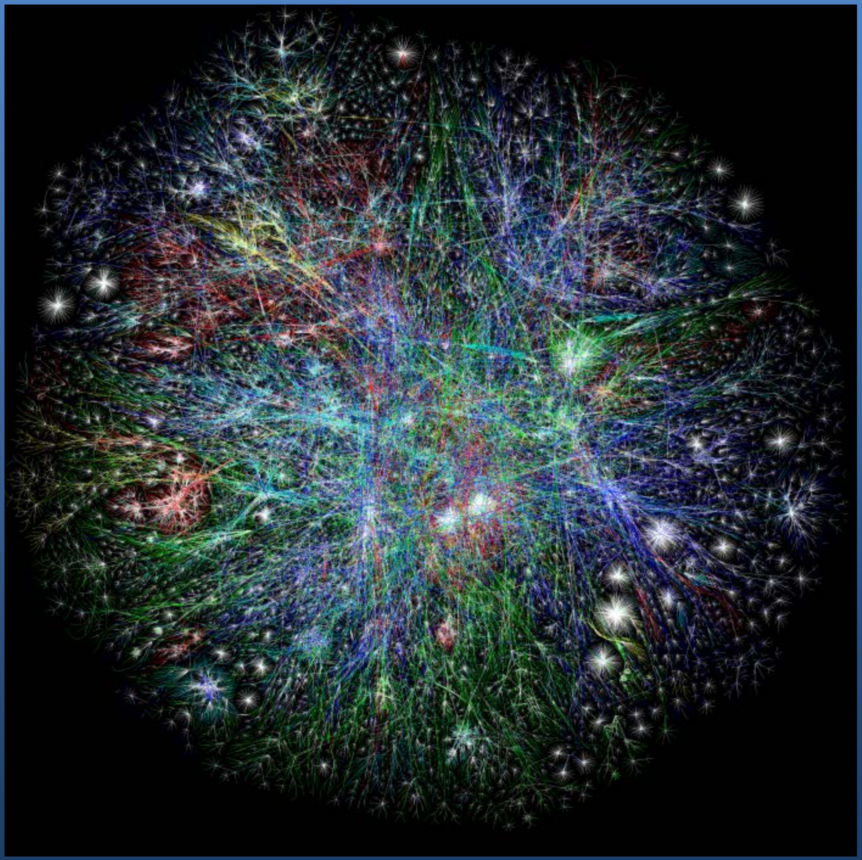
IEEE TAC 2017

S. Formentin, FD, R. Tempo, L. Zaccarian and S. M. Savaresi, "Robust Linear Static **Anti-Windup with Probabilistic Certificates**"

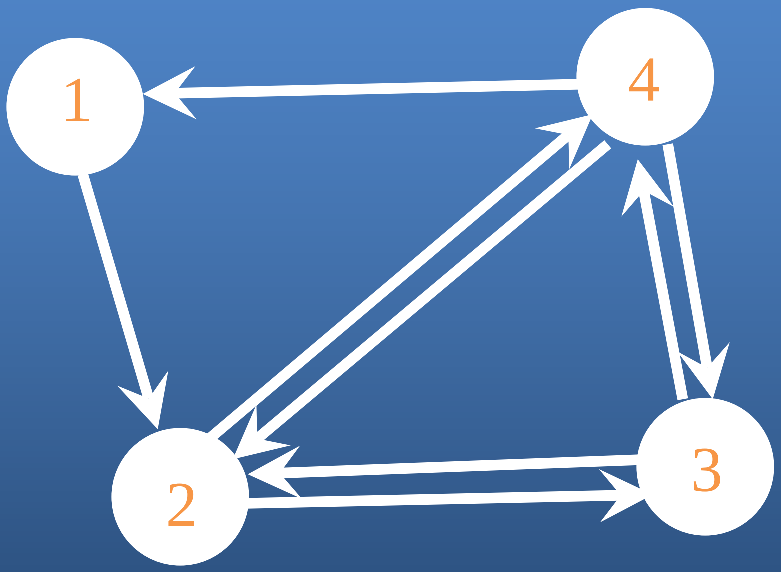
IEEE TAC 2017

Beyond classical robustness: Randomized PageRank computation



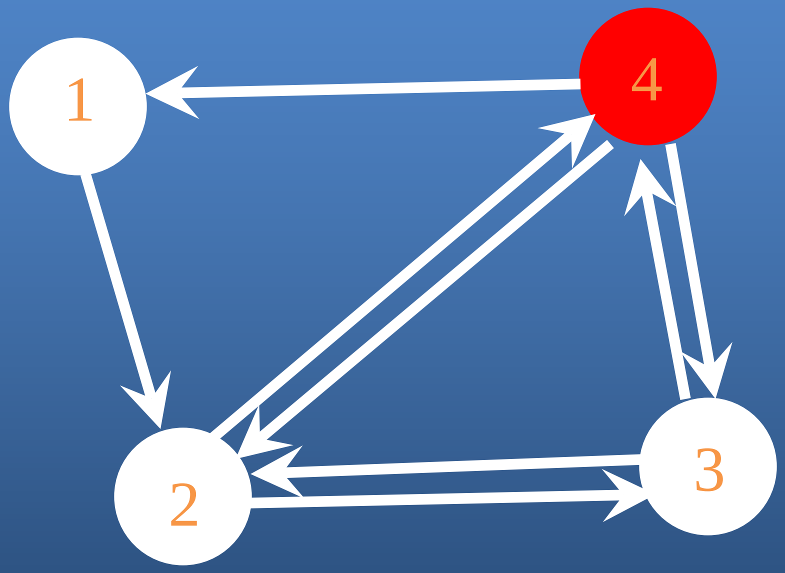


Randomized PageRank



Local communication protocol: at time k randomly select page i for PageRank update

Randomized PageRank



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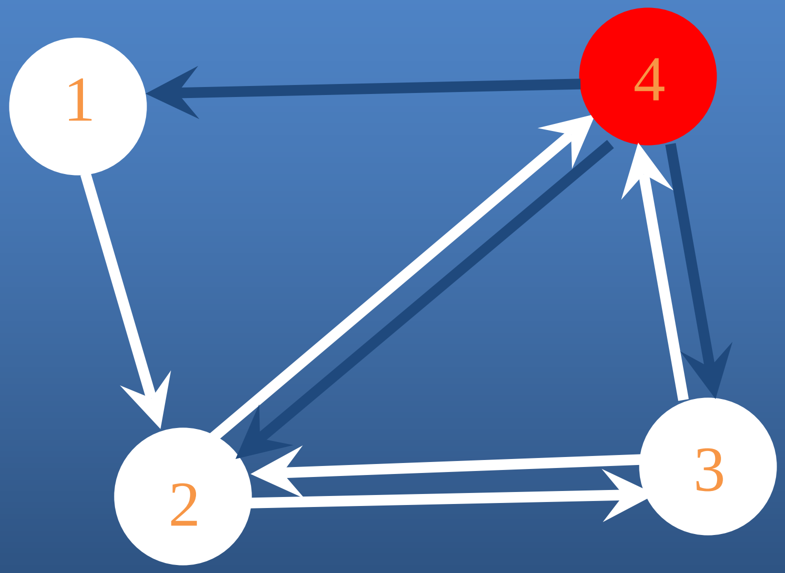
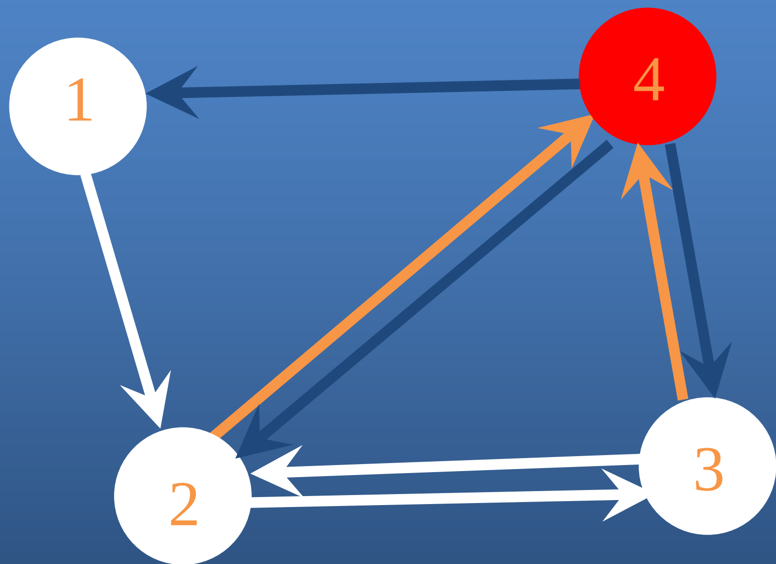


Local communication protocol: at time k randomly select page i for PageRank update

1. send PageRank value of page i to the outgoing pages

Randomized PageRank



Local communication protocol: at time k randomly select page i for PageRank update

1. send PageRank value of page i to the outgoing pages

2. request PageRank values from incoming pages

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- H. Ishii and R. Tempo, “Distributed Randomized Algorithms for the PageRank Computation,” IEEE TAC 2010

- H. Ishii and R. Tempo, “The PageRank Problem, Multi-Agent Consensus and Web Aggregation: A Systems and Control Viewpoint,” IEEE CSM 2014
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- H. Ishii, R. Tempo and E. W. Bai, “A Web Aggregation Approach for Distributed Randomized PageRank Algorithms,” IEEE TAC 2012
- H. Ishii and R. Tempo, “Distributed Probabilistic Approach for the PageRank Algorithm,” J. SICE 2011
- H. Ishii and R. Tempo, “Distributed Randomized Algorithms for the PageRank Computation,” IEEE TAC 2010

CSM Best Paper Award

From networked systems to networked uncertain systems

Thanks to

T. Alamo, F. Allgower, E.W. Bai, B.R. Barmish, T. Basar, G. Calafiore, E.F. Camacho, M.C. Campi, E. Capello, M. Chamanbaz, P. Colaneri, F. Dabbene, S. Formentin, P. Frasca, Y. Fujisaki, S. Garatti, H. Ishii, C. Lagoa, M. Lorenzen, Y. Oishi, S. Parsegov, D. Peaucelle, B.T. Polyak, M. Prandini, A. Proskurnikov, L. Qiu, C. Ravazzi, M. Sznaier, M. Vidyasagar, T. Wada, K. You, L. Zaccarian


Thank you