Dinamiche di opinioni: un po' di testardaggine non fa mai male

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Opinion dynamics

Assumptions:

- Population of individuals
- Individuals have opinions
- People interact and opinions evolve

Main challenges:

1. Modeling: description of social behavior
2. Analysis: extract low-dimensional features
3. Control: mechanisms to influence the final opinions

Roberto's activity and merits

Towards a unified framework for everybody

1. Reducing the gap between social network analysis and control
   - New dynamical models for opinion formation (2012-2016)
     - opinion dynamics with stubborn agents
     - asynchronous models with pair-wise interactions
     - multidimensional models with interdependent topics
   - Beyond PageRank (2016):
     - wider class of centrality measures
     - missing links between PageRank and opinion dynamics
   - Identification methods for social influence (2016-2017)

2. Sharing results with broad research community
   - *A. V. Proskurnikov, R. Tempo*, A tutorial on modeling and analysis of dynamic social networks. Part I., Annual Reviews in Control, 2017
   - *P. Frasca, H. Ishii, C. Ravazzi and R. Tempo*, Distributed Randomized Algorithms for Opinion Formation, Centrality Computation and Power Systems Estimation: A Tutorial Overview, European Journal of Control, 2015

Network Science on Belief Systems

Science, Oct. 2016

N. E. Friedkin, A. V. Proskurnikov, R. Tempo and S. E. Parsegov, Network Science on Belief System Dynamics under Logic Constraints, Science, Oct. 2016

Preliminaries

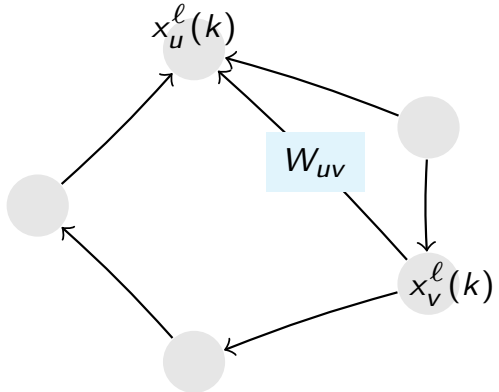
Assumptions:

- Population of individuals
- Discussion on several topics
- Individuals have opinions
- People interact
- Opinions evolve

Mathematical model:

$\mathcal{G} = (\mathcal{V}, \mathcal{E}, W) \leftrightarrow$ Social network

- $v \in \mathcal{V} \leftrightarrow$ agents
- $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V} \leftrightarrow$ interactions
- $W \in \mathbb{R}^{\mathcal{V} \times \mathcal{V}} \leftrightarrow$ influences
- $W_{uv} = 0$ if $(u, v) \notin \mathcal{E}$
- $x_v^\ell(k) \in \mathbb{R} \leftrightarrow$ opinions on issue ℓ
- $x(k+1) = f_W(x(k))$



How opinions evolve ($x(k)$) based on interpersonal relations (W), external influences and a priori prejudices (f)?

Assumptions: collaboration with prejudices [Friedkin & Johnsen, 1999]

$$x_i(0) = u_i$$
$$x_i(k+1) = \lambda_i \sum_j W_{ij} x_j(k) + (1-\lambda_i) u_i$$

- $W_{ij} \in [0,1] \leftrightarrow$ strength of interactions/influences

$$W_{ij} = 0 \text{ if } (i,j) \notin \mathcal{E} \qquad \sum_{j \in \mathcal{V}} W_{ij} = 1$$

- $\lambda_i = 1 - W_{ii} \leftrightarrow$ sensitivity to the opinions of the others

(if $\lambda_i = 0$, i totally stubborn; if $\lambda_i = 1$, i open minded)

- $u_i \in \mathbb{R} \leftrightarrow$ persistent prejudices (input at every time step)

Convergence?

Oblivious agent = agent that is neither stubborn nor influenced by a stubborn agent

- Presence of oblivious agents is the only reason for instability
- Sufficient condition for convergence: **no oblivious agents**

$$x^\star = \lim_{k \to +\infty} x(k) = (I - \Lambda W)^{-1}(I - \Lambda)u, \quad \Lambda = \text{diag}(\lambda)$$

- complex limit opinion profiles (no consensus)
- limit opinion of each agent is a convex combination of prejudices

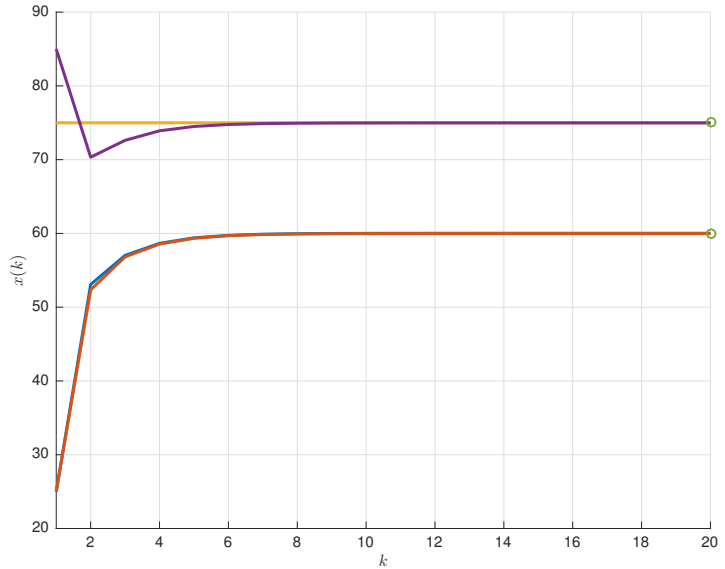
Example by F&J (1998)

$$u = [25\ 25\ 75\ 85]^\top$$

$$x = [60\ 60\ 75\ 75]^\top$$

$$W = \begin{bmatrix} .220 & .120 & .360 & .300 \\ .147 & .215 & .344 & .294 \\ 0 & 0 & 1 & 0 \\ .090 & .178 & .446 & .286 \end{bmatrix}$$

$$\Lambda = \mathrm{diag}(.780, .785, 0, .714)$$



Result I
Randomization and Gossip interactions

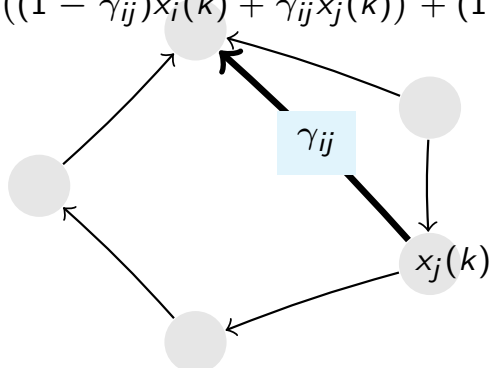
"It is obvious that interpersonal influences do not occur in the simultaneous way", [Friedkin and Johnsen, 1999]

Assumptions:

- interactions occur at random times
- gossips: pairwise random interactions

New model:[1] At time k, directed link (i, j) uniformly sampled from $\mathcal{E}$

$$x_i(k+1) = h_i\big((1-\gamma_{ij})x_i(k) + \gamma_{ij}x_j(k)\big) + (1-h_i)u_i$$

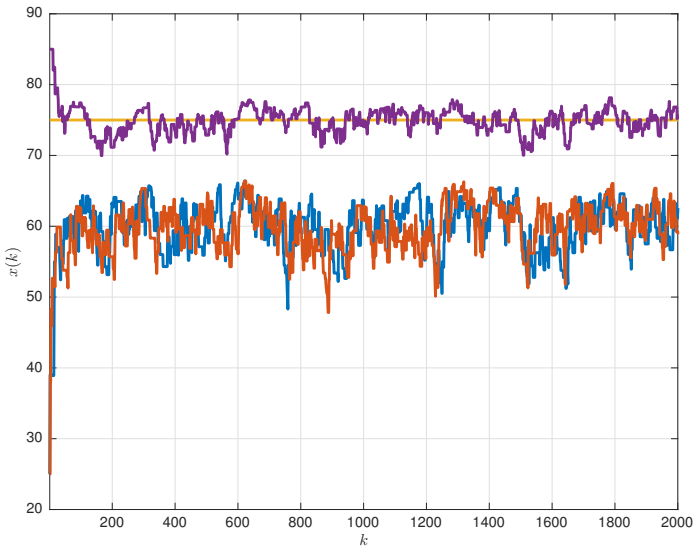


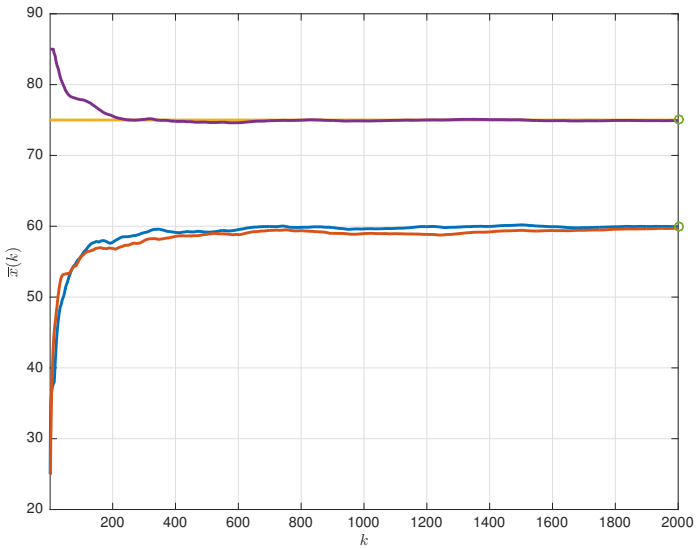
Convergence?

[1] P. Frasca, C. Ravazzi, R. Tempo, H. Ishii, 2014

F&J opinion dynamics

Randomization and gossips





$x(k)$ persistently oscillates but... is ergodic! [2]

- $\overline{x}(k) = \frac{1}{k+1} \sum_{\ell=0}^{k} x(\ell) \to \mathbb{E}[x_\infty]$
- there exist $h_i = h_i(\Lambda_{ii}), \gamma_{ij} = \gamma_{ij}(\Lambda_{ii}, W_{ij})$ s.t. $\mathbb{E}[x_\infty] = x^\star$

[2] C. Ravazzi, P. Frasca, R. Tempo and H. Ishii, 2015

Result II
Multidimensional models

Assumptions

- agents discuss m topics
- topics are correlated

New model[3] :

$$x_i(0) = u_i \in \mathbb{R}^m$$

$$x_i(k+1) = \lambda_i C \sum_j W_{ij} x_j(k) + (1 - \lambda_i) u_i$$

- $u_i \leftrightarrow$ prejudices of agent i
- $W \leftrightarrow$ strength of interactions
- $\lambda_i \leftrightarrow$ sensitivity of agent i to the opinions of the others
- $C \in \mathbb{R}^{m \times m} \leftrightarrow$ multi-issues dependence structure

[3] S. E. Parsegov, A. V. Proskurnikov, R. Tempo and N. E. Friedkin, 2016

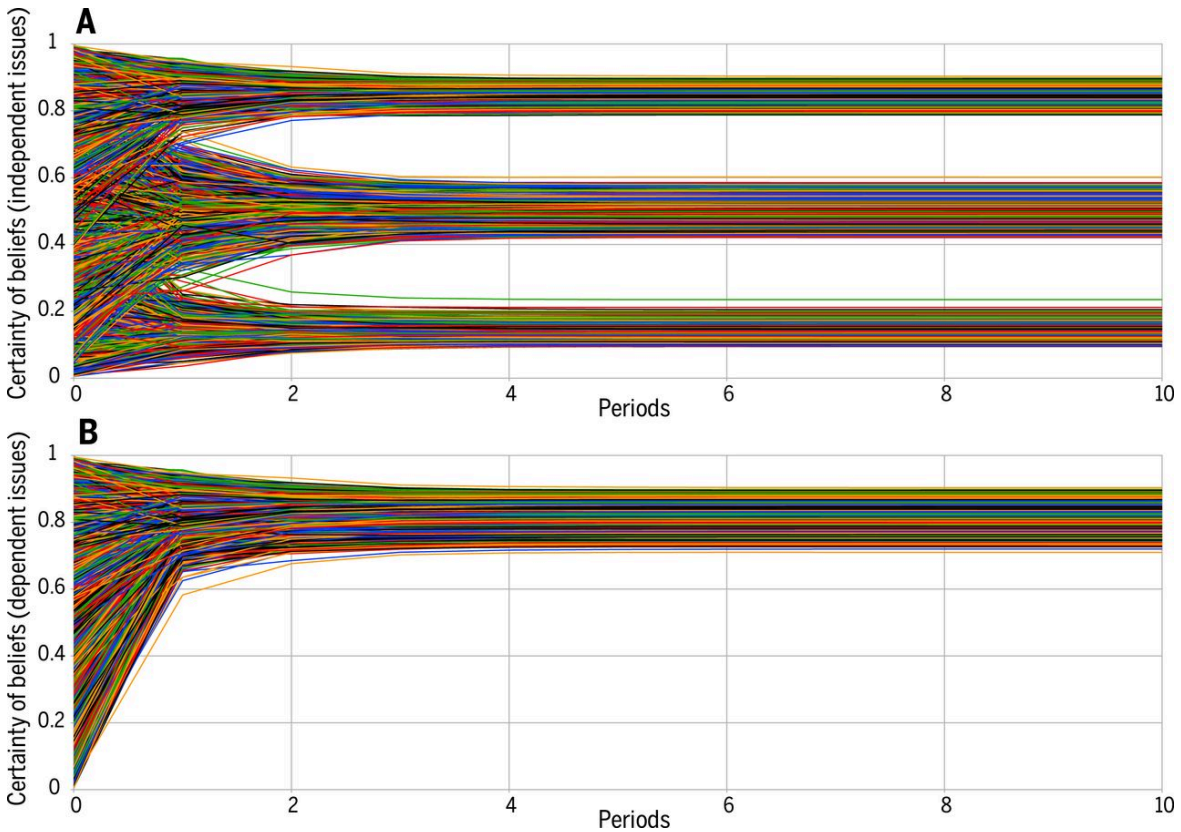
Case of study:

- Network model of USA population: sparse random structure
- Group of individuals attentive to three statements
  a) Saddam Hussein has a stockpile of biological and chemical weapons of mass destruction (WMD)
  b) Saddam Hussein supports Osama bin Laden's terroristic attacks threat to the USA
  c) Preemptive invasion of Iraq is a "just war"

Colin Powell speech to the UN Security Council (Feb. 2003)
Removal of WMD from Iraq will break the alliance of Iraq and Al-Qaeda and it will bring Iraq into compliance with the UN Security Council disarmament Resolution 1441

F&J opinion dynamics

2003 Iraq War – Three Key Topics



A. $C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **B.** $C = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

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[4] N. E. Friedkin, A. V. Proskurnikov, R. Tempo and S. E. Parsegov, 2016

Result III
Social influence estimation from data

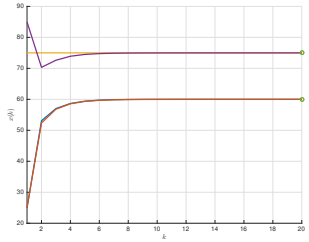
F&J opinion dynamics

Social influence estimation

Network system:

$\mathcal{G} = (\mathcal{V}, \mathcal{E}, W)$

Model:



$$x^j(k+1) = \Lambda W x^j(k) + (I - \Lambda) u^j$$

Observations:

m topics $\longrightarrow$

$$X(\infty) = [x^1(\infty), \ldots, x^m(\infty)]$$

final opinions

Goal: Given initial opinions $U = [u^1, \ldots, u^m]$, final opinions $X(\infty) = [x^1(\infty), \ldots, x^m(\infty)] \Longrightarrow$ Identify W

$$m \ll |\mathcal{V}| \Longrightarrow \textbf{ill-posed problem!!!}$$

Assumption:

W sparse $\leftrightarrow$ people are influenced by few friends

Inverse problem: cast as a compressed sensing problem

$$\min_{w_\ell} \|w_\ell\|_1 \qquad \text{s.t.} \quad \underbrace{X(\infty)^\top w_\ell = b_\ell}_{\text{steady state conditions}} \quad \underbrace{\mathbb{1}^\top w_\ell = 1}_{\text{stochasticity}} \quad \underbrace{w_\ell \geq 0}_{\text{positivity}}$$

- $d \ll n$ number of non-zeros in each row
- $X(\infty)^\top \in \mathbb{R}^{m \times |\mathcal{V}|}$ fat sensing matrix ($m \ll |\mathcal{V}|$)
- $b_\ell = b_\ell(X(\infty), \Lambda, U) \in \mathbb{R}^m$ constant term

Sufficient conditions for reconstruction [5]

- synchronous dynamics: no oblivious agents, gaussian initial opinions, W symmetric, aperiodic and irreducible $\Longrightarrow m = O(\frac{(1+\lambda)^2}{(1-\lambda)^2} d \log |\mathcal{V}|)$
- gossip random interactions: $m = O(\frac{(1-\lambda+\lambda/d)^2}{(1-\lambda)^2} d \log |\mathcal{V}|)$

[5] C. Ravazzi, R. Tempo, F. Dabbene, 2017

Concluding remarks

- Building a unified framework to analyze belief systems
- Dissemination of results to broad community
- Results on
  - modeling: systems with stubborn agents
  - analysis: centrality measures to detect influential leaders and randomized algorithms for PageRank
  - identification: modern tools for estimation of social influence

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